2019自由电子气输运性质

2016年4月26日 16:07

电子是固体物理的主角.(晶格是舞台,声子是配角)

金属中的自由电子气:传输电荷和热量.

(I) 金属电导的 Drude-Sommerfeld 理论

✓ 忽略电子-电子及电子-离子实的长程库仑相互作用

✓ 只考虑瞬时碰撞

⊛ 弛豫时间为 τ(一次碰撞到下一次平均时间),费米面附近 τ_F.

(1) Electric Conductivity

建立运动方程

$$[\vec{p}(\delta t)-\vec{p}(0)] = -\frac{\delta t}{\tau}\vec{p}(0) + \vec{F}\cdot\delta t$$

$$\frac{\partial \vec{p}}{\partial t} = -\frac{\vec{p}}{\tau} + \vec{F} \quad \text{E.O.M}$$

since $\vec{F} = -e\cdot\vec{E}$.
and for steady state $\frac{\partial p}{\partial t}=0$

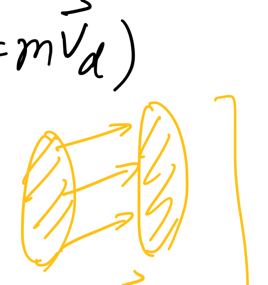
(Drifting velocity) 漂移速度

$$\Rightarrow \frac{\vec{p}}{\tau} = \vec{F} \Rightarrow \vec{v}_d = -e\vec{E}\tau/m \quad (\vec{p}=m\vec{v}_d)$$

[classical picture]

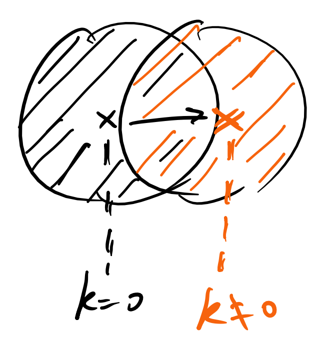
$$\vec{J} = -n\cdot e\cdot\vec{v}_d = -ne\cdot\frac{-e\vec{E}\tau}{m} = \frac{ne^2\tau}{m}\cdot\vec{E}$$

$$\Rightarrow \sigma = ne^2\tau/m$$



$\vec{I} = -ne\vec{v}_d\cdot S$
$\Rightarrow \vec{j} = -ne\vec{v}_d$

[Sommerfeld's picture] $\hbar\frac{d\vec{k}}{dt} = \vec{F}$



Relaxation time is τ

$$\hbar\vec{k}(\tau) - \hbar\vec{k}(0) = -e\vec{E}\cdot\tau \quad (\hbar\vec{k}(0)=0)$$

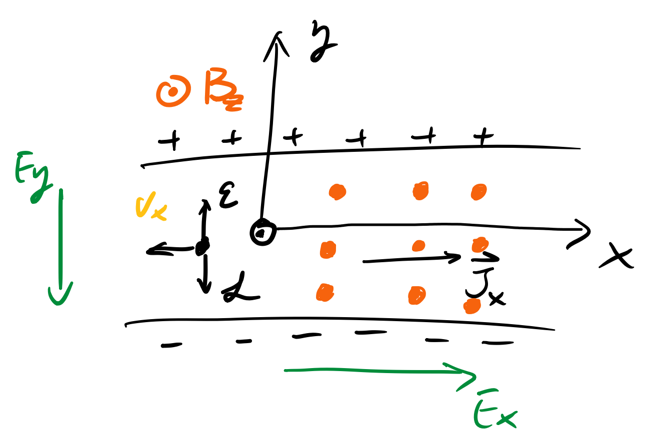
$$\Rightarrow \hbar\vec{k}(\tau) = -e\vec{E}\tau \; (= m\vec{v}_d)$$

Drifting velocity $\vec{v}_d = -\frac{e\vec{E}\tau}{m}$ (耗散-驱动力抵消)

$$\vec{J} = \sigma\vec{E} \Rightarrow \sigma = ne^2\tau/m$$

(2) Hall Effect

X-Y方向运动方程



$$X: \frac{dp_x}{dt} = -\frac{p_x}{\tau} - eE_x$$

$$Y: \frac{dp_y}{dt} = -\frac{p_y}{\tau} - e(E_y - \frac{p_x}{m}B_z)$$

$f = -eE_x - e\cdot\vec{v}_x\times\vec{B}_z$

Steady state (稳态) $dp/dt = 0$

$$\Rightarrow \begin{cases} \frac{p_x}{\tau} = -eE_x \\ \frac{p_y}{\tau} = -eE_y + e\frac{p_x}{m}B_z = 0 \end{cases} \text{(已知无横向运动)}$$

$$\Rightarrow \begin{cases} p_x = -eE_x\tau \\ E_y = \frac{p_x}{m}B_z \end{cases}$$

外加磁场 不改变纵向电阻.

$$J_x = -ne\cdot v_x = -ne\frac{p_x}{m} \left(= ne\cdot\frac{eE_x\tau}{m} = \frac{ne^2}{m}E_x\tau\right)$$

$$\Rightarrow E_y = \frac{p_x}{m}B_z = -\frac{J_x}{ne}\cdot B_z = R_H\cdot J_x\cdot B_z.$$

(i) Hall coefficient

$$R_H = \frac{E_y}{J_x\cdot B_z} = -\frac{1}{ne}$$

✓ 物理过程

施加纵向(x方向)电压,产生纵向电流)

施加 z方向磁场,电子感受负y方向洛伦兹力,向纸侧累积产生正y电场

横向洛伦兹力与横向电场施加静电力抵消,电流沿x方向流动.

(ii) 电阻率

建立一般运动方程:

$$\begin{cases} \frac{dp_x}{dt} = -\frac{p_x}{\tau} - e(E_x + \frac{p_y}{m}\cdot B_z) \\ \frac{dp_y}{dt} = -\frac{p_y}{\tau} - e(E_y - \frac{p_x}{m}\cdot B_z) \end{cases}$$

$$\Rightarrow \begin{cases} \frac{p_x}{\tau} + \frac{eB}{m}p_y = -eE_x \\ -\frac{eB}{m}p_x + p_y/\tau = -eE_y \end{cases}$$

引入 $\omega_c = \frac{eB}{m}$, $p_x = -J_x\cdot\frac{m}{ne}$, $p_y = -J_y\cdot\frac{m}{ne}$

$$\Rightarrow \begin{cases} J_x(-\frac{m}{ne\tau}) + \omega_c\cdot J_y(-\frac{m}{ne}) = -eE_x \\ -\omega_c J_x(-\frac{m}{ne}) + J_y(-\frac{m}{ne\tau}) = -eE_y \end{cases}$$

$$\Rightarrow \begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix}\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \sigma_0\begin{pmatrix} E_x \\ E_y \end{pmatrix} \quad \left[\sigma_0 = \frac{ne^2\tau}{m}\right]$$

电阻率:

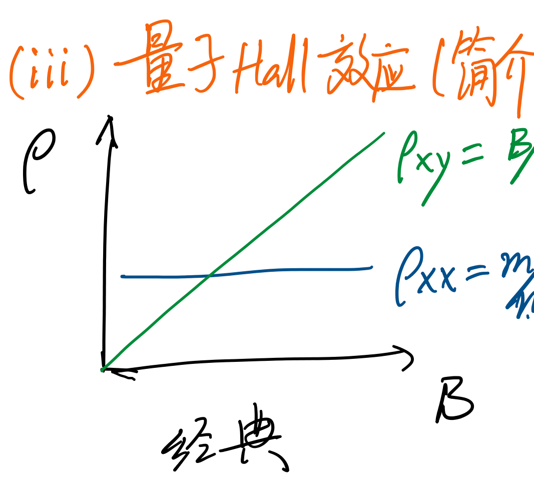
$$\rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{xx} \end{pmatrix},$$ 其中 $\rho_{xx} = 1/\sigma_0 = m/ne^2\tau$

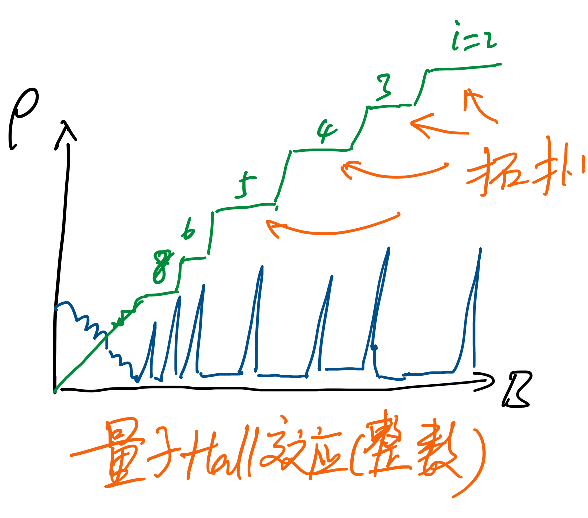
$\rho_{xy} = \omega_c\tau/\sigma_0 = \frac{eB}{m}\cdot\tau\cdot\frac{m}{ne^2\tau} = \frac{B}{ne}$

取倒数 电导率 小习题(写出 σ_{xx} 与 σ_{xy})

$$\sigma = \sigma_0\cdot\begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix}^{-1} = \sigma_0\cdot\frac{1}{1+\omega_c^2\tau^2}\cdot\begin{pmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1 \end{pmatrix}$$

(iii) 量子Hall效应(简介)





QHE: $\rho_{xx} = 0$

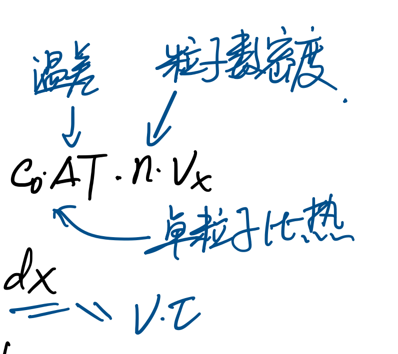
$$\rho_{xy} = \frac{2\pi\hbar^2}{e^2}\frac{1}{\nu} \leftarrow \nu\text{为整数}$$

$$\begin{pmatrix} 0 & \rho_{xy} \\ -\rho_{xy} & 0 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 0 & -1/\rho_{xy} \\ 1/\rho_{xy} & 0 \end{pmatrix}$$
$$\rho \Leftrightarrow \sigma$$

$$\sigma_{xy} = \frac{e^2}{2\pi\hbar^2}\cdot\nu$$ 拓扑数.

(II) 自由电子热导

(1) 扩散型热流-温度梯度关系



$$\vec{J}_Q = -\kappa\vec{\nabla}T$$ (κ: 热导)

热导 $\kappa = \kappa_e + \kappa_{ph}$ (κ_e: 电子热导, κ_{ph}: 声子热导)

$$\kappa_e = \frac{1}{3}C\cdot v\cdot l$$

(C: 等容比热, v: 平均速度, l: 平均自由程.)

粒子流 $j_x = v_x n$, 携带热量 $c_0\Delta T\cdot n\cdot v_x$ (ΔT: 温差, n: 粒子数密度, c_0: 单粒子比热)

$$\Rightarrow J_Q = c_0\cdot n\cdot v_x\cdot\Delta T = C_V\cdot v_x\cdot\frac{dT}{dx}\cdot dx \quad (c_0 n = C_V,\; dx = v\cdot\tau)$$

$$= C_V v_x^2\tau\cdot dT/dx$$

$$= C_V v^2/3\cdot\tau\; dT/dx$$

$$= \frac{1}{3}C_V v\cdot l\cdot dT/dx$$

($v_x^2 = \frac{1}{3}v^2$ 平均速度) 平均自由程 ($l = v\cdot\tau$)

按照定义 $\kappa_e = \frac{1}{3}C_V v\cdot l$.

结合自由电子气的比热 $C_V = \frac{\pi^2}{2}nk_B T/T_F$

$$\Rightarrow \kappa_e = \frac{\pi^2}{6}nk_B\frac{T}{T_F}\cdot v_F\cdot l_F$$

$$\kappa_e = \frac{\pi^2}{6}nk_B T\cdot\frac{k_B}{\frac{1}{2}mv_F^2}\cdot v_F\cdot v_F\tau_F$$

$(T_F = \frac{1}{2}mv_F^2/k_B)$

$$\kappa_e = \frac{\pi^2 nk_B^2\tau_F}{3m}\cdot T \quad \text{i.e., } \kappa_e\propto T$$

(2) Wiedemann-Franz 定律

☆ 在不太低的温度下,自由电子金属的热导和电导之比 $\propto T$

电导: $\sigma_0 = \frac{ne^2\tau_F}{m}$, $\kappa_e = \frac{\pi^2 nk_B^2\tau_F}{3m}\cdot T$

$$\Rightarrow \kappa_e/\sigma_0 = \frac{\pi^2 nk_B^2\tau_F}{3m}\cdot T \Big/ \frac{ne^2\tau_F}{m} = \left(\frac{\pi^2 k_B^2}{3e^2}\right)\cdot T$$

(洛伦兹数 L)

$L = 2.45\times10^{-8}\,W\cdot\Omega/K^2$ (小习题)

[其中 $k_B = 1.38\times10^{-23}J/K$, $e = 1.6\times10^{-19}C$, $(J/C)^2 = V^2 = W\cdot\Omega$]

(III) 自由电子气理论的缺陷.

① 可以解释相当一部分金属的平衡与输运性质,但远不是全部

例如过渡族金属(e.g. 铁)和稀土金属等(3d,4f局域电子)

② 无法解释正Hall系数 $(-\frac{1}{ne}) > 0$?! 也无法解释磁电阻 (magnetoresistence)

③ 无法解释金属具有磁性,超导等

④ 需要统一理论框架理解金属和绝缘体 ⇒ 能带论!