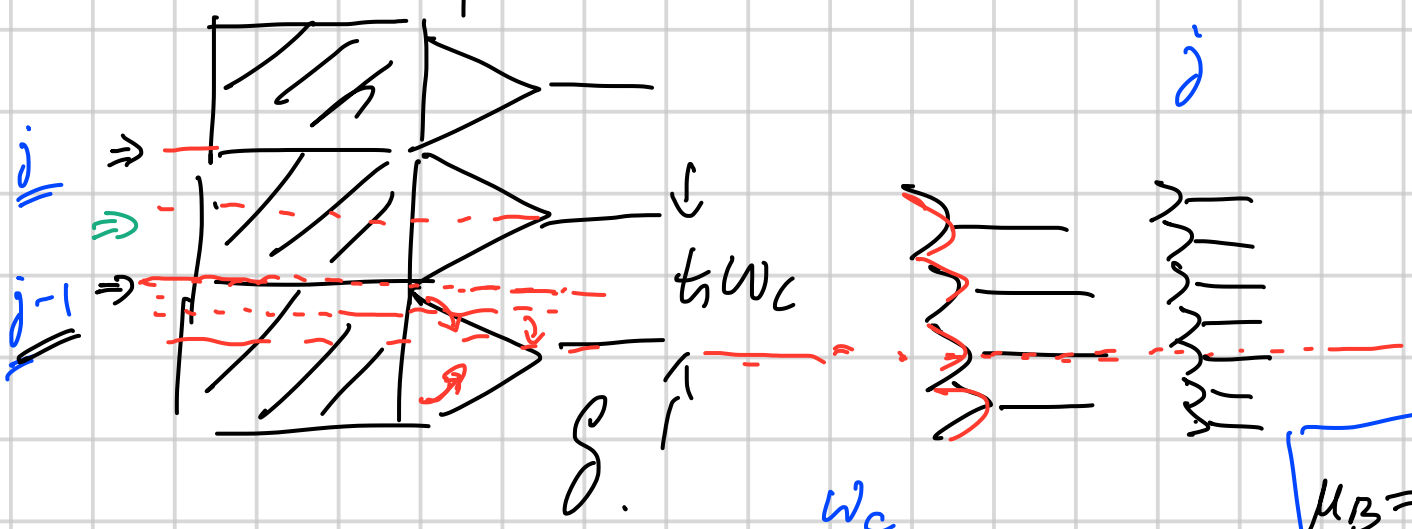
## ② 德哈斯-范阿尔芬效应（量子振荡）




(i) 系统能量 $E = \frac{1}{2}\hbar\left(\frac{eB}{mc}\right)N = N\cdot\mu_B\cdot B$. （大B极限）

→ 全部电子填充在 $j=0$ 能级上

LLL

(ii) 减小磁场，系统能量E下降（$E\propto B$）直至 $g=N$

此后，电子开始填充 $j=1$ 能级.

(iii) 一般地 $j\cdot g < N < (j+1)g$ 时

即 $j\cdot\frac{2e}{hc}B\cdot L^2 < N < (j+1)\cdot\frac{2e}{hc}B\cdot L^2$

此时系统能量：

$\varepsilon_j = \hbar\omega_c\left(j+\frac{1}{2}\right)$

$$E_0(B) = \sum_{i=0}^{j-1}\varepsilon_i\cdot g + (N-g\cdot j)\cdot\varepsilon_j$$

$$= g\cdot\hbar\omega_c\cdot\frac{\frac{1}{2}+(j-1)+\frac{1}{2}}{2}\cdot j + (N-gj)\cdot\hbar\omega_c\left(j+\frac{1}{2}\right)$$

$g = \frac{2e}{hc}BL^2$

$$= g\cdot\frac{1}{2}\hbar\omega_c\cdot j^2 + N\hbar\omega_c\left(j+\frac{1}{2}\right) - gj\hbar\omega_c\left(j+\frac{1}{2}\right)$$

$$= N\hbar\omega_c\left(j+\frac{1}{2}\right) - g\cdot j(j+1)\cdot\frac{1}{2}\hbar\omega_c$$

$$= N\cdot\mu_B\cdot B\cdot(2j+1) - g\cdot j(j+1)\cdot\mu_B\cdot B = \mu_B\cdot B[(2j+1)N - (j+1)jg]$$

(iv) 讨论：(a) $j\cdot g = N \Rightarrow E_1 = \mu_B\cdot B_1\, j\cdot N$.

(b) $(j+1)g=N \Rightarrow E_2=\mu_B\cdot B_2(j+1)N$.

$$\begin{cases} j\cdot\frac{2e}{hc}\cdot B_1\cdot L^2=N \\ (j+1)\frac{2e}{hc}B_2\cdot L^2=N \end{cases} \Rightarrow \begin{cases} 1/B_1=j\cdot\frac{2e}{hc}\cdot\frac{L^2}{N} \\ 1/B_2=(j+1)\cdot\frac{2e}{hc}\cdot\frac{L^2}{N} \end{cases}$$

$$\begin{cases} E_1=\mu_B\cdot N\cdot j\cdot\frac{hc}{2e}\frac{N}{L^2}\cdot\frac{1}{j}=\mu_B\cdot\frac{N^2}{L^2}\cdot\frac{hc}{2e} \\ E_2=\mu_B\cdot(j+1)N\cdot\frac{hc}{2e}\cdot\frac{N}{L^2}\cdot\frac{1}{j+1}=\mu_B\frac{N^2}{L^2}\cdot\frac{hc}{2e} \end{cases}$$

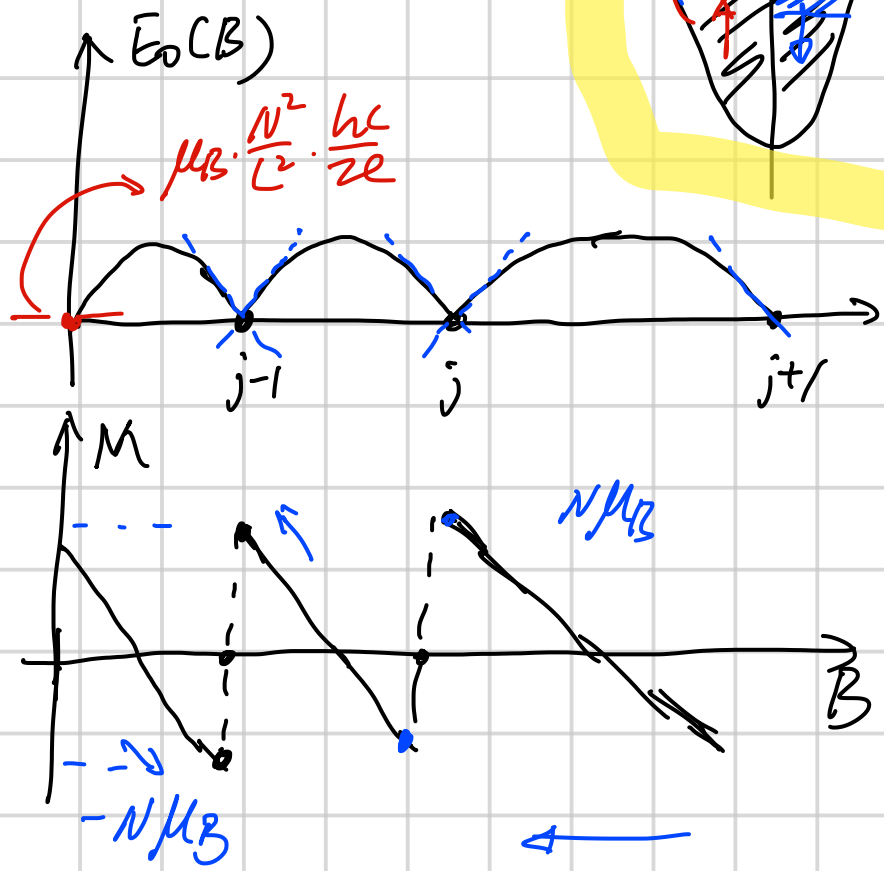
$\Rightarrow E_1=E_2$

(v) 磁矩: $M=-\dfrac{\partial E_0(B)}{\partial B}$

$M=-N\mu_B(2j+1)+2(j+1)j\mu_B\cdot g$.

当 $j\cdot g=N$ 时 $M=N\cdot\mu_B$

$(j+1)g=N$ 时 $M=-N\mu_B$




周期:

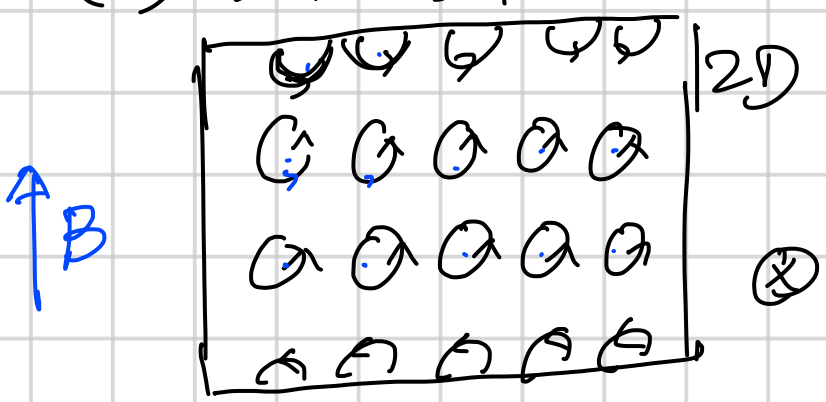
$$\Delta\left(\frac{1}{B}\right)=\left(\frac{1}{B_2}-\frac{1}{B_1}\right)=\frac{2e}{hc}\cdot\frac{L^2}{N}\quad \text{const}$$

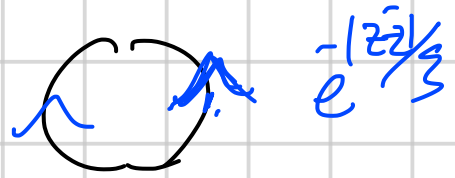
(vi) dHvA效应: 磁场随外磁场周期地反转, $\Delta(1/B)\sim$ const.

$\Rightarrow \Delta\left(\frac{1}{B}\right)=\frac{2e}{hc}\cdot\frac{L^2}{N}\left(\sim 2\pi R^2\sim\frac{2hc}{eB}\sim\frac{1}{A}\right)$ 反比于费米面面积.

费米面形状


(vii) 量子Hall效应.




体态导电. 局域态

chiral edge